

# Scalable Modeling and Performance Evaluation of Dynamic RED Router using Fluid-Flow Approximation

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## ABSTRACT

In recent years, AQM (Active Queue Management) mechanisms, which support the end-to-end congestion control mechanism of TCP (Transmission Control Protocol), have been widely studied in the literature. AQM mechanism is a congestion controller at a router for suppressing and stabilizing its queue length (i.e., the number of packets in the buffer) by actively discarding arriving packets. Although a number of AQM mechanisms have been proposed, behaviors of those AQM mechanisms other than RED (Random Early Detection) have not been fully investigated. In this paper, using fluid-flow approximation, we analyze steady state behavior of DRED (Dynamic RED), which is designed with a control theoretic approach. More specifically, we model several network components such as congestion control mechanism of TCP, DRED router, and link propagation delay as independent SISO (Single-Input Single-Output) continuous-time systems. By interconnecting those SISO models, we obtain a continuous-time model for the entire network. Unlike other fluid-based modeling approaches, our analytic approach is scalable; our analytic approach is scalable in terms of the number of TCP connections and DRED routers since both input and output of all continuous-time systems are uniformly defined as a packet transmission rate. By performing steady-state analysis, we derive TCP throughput, average queue length of DRED router, and packet loss probability. Through several numerical examples, we quantitatively show that DRED has an intrinsic problem in high-speed networks; i.e., DRED cannot stabilize its queue length when the bottleneck link bandwidth is high. We also validate accuracy of our analytic approach by comparing analytic results with simulation ones.

**Keywords:** Fluid-Flow Approximation, Active Queue Management, DRED (Dynamic RED), Performance Evaluation, TCP (Transmission Control Protocol)

## 1. INTRODUCTION

In recent years, active queue management mechanisms which support the end-to-end congestion control of TCP (Transmission Control Protocol) have been widely studied.<sup>1</sup> Active queue management is a mechanism for congestion control at a router, which controls its queue length (the number of packets in its buffer) by dropping packets early before buffer overflow occurs. It is capable of decreasing the number of dropped packets as well as stabilizing the queue length of a router at a smaller level. Thereby, it has benefits such as the reduction in end-to-end packet transmission delays. RFC2309<sup>1</sup> recommends the use of RED (Random Early Detection), which is a representative active queue management mechanism. One of the goals of RED is to stabilize the average queue length of a router at a smaller level. RED randomly drops packets, which arrive at a router, with a probability computed from the average queue length of the router. However, it is known that great care is required to configure RED parameters to achieve good performance under different network conditions.

Accordingly, a large number of methods have been proposed, including improved versions of RED and new active queue management mechanisms. Some literature<sup>2-5</sup> proposed new active queue management algorithms designed by ad hoc methods. Other literature<sup>6, 7</sup> proposed active queue management mechanisms based on control theory, taking account of the fact that TCP is a feedback congestion control. For example, REM (Random Exponential Marking)<sup>6</sup> is based on a classical control theory, PI (Proportional-Integral) control. In the REM algorithm, the router is a controller, the average queue length of the router is an input, and the packet drop probability is an output. REM determines packet drop rate on the basis of price: i.e., (queue length - target queue length) + (packet drop rate - line speed). DRED (Dynamic RED)<sup>7</sup> is based on another classical control theory, I (Integral) control. In the DRED algorithm, the router is a controller, the average queue length of the router is an input, and the packet drop probability is an output. DRED can adjust the average queue length of the router to the predetermined target value. The literature<sup>7</sup> demonstrated by simulation that the control parameters of

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DRED,  $\alpha$  and  $L$ , affected the steady state characteristics of the router such as the average queue length and the packet drop probability. However, the steady state behavior of DRED has not been fully investigated.

At the same time, several studies on the modeling of active queue management mechanisms using fluid-flow approximation are being conducted.<sup>8-11</sup> The literature<sup>8</sup> modeled each of the congestion control mechanisms of TCP and the active queue management mechanism RED as a separate fluid-flow approximation model. It also compared the analysis results of fluid-flow approximation models with the results of ns2 simulation to validate the approximation models. The literature<sup>9</sup> analyzed the local stability of RED by linearizing the fluid-flow approximation model proposed in the literature<sup>8</sup> around the equilibrium point. The literature<sup>10</sup> extended the stability analysis in the above literature<sup>9</sup> so as to derive the conditions to stabilize the performance of a network with different propagation delays of TCP connections. The literature<sup>11</sup> obtained a method to analyze a large-scale network using fluid-flow approximation. It demonstrated that the entire closed-loop network with cascading connections can be modeled by modeling each network component as a continuous-time system where the input and output were standardized to match the packet transfer rate. However, these studies only dealt with RED among active queue management mechanisms and none of the others have been thoroughly modeled or evaluated for their performance.

Therefore, in this paper we analyze the steady state behavior of the active queue management mechanism DRED by using a mathematical analytical method, fluid-flow approximation. In particular, we first model DRED as a continuous-time system, using the modeling method proposed in the literature.<sup>11</sup> We then build a model of an entire network by interconnecting the DRED model with the model of the TCP congestion control mechanism and that of link propagation delay suggested in the literature.<sup>11</sup> We represent the model of the entire network as a block diagram using MATLAB/Simulink,<sup>12</sup> which is a control system CAD tool, and thereby analyze the steady state behavior of DRED. Specifically, we measure the quantitative impact of the number of TCP connections and the bandwidth of bottleneck links on the steady state characteristics such as average queue length and packet drop probability of the router. We also examine the validity of the approximate analytical method by comparing the results of numerical simulations using this analytical model with those of discrete-time simulations using ns-2.<sup>13</sup>

The structure of this paper is as follows. First, Section 2 provides an overview of the active queue management mechanism DRED as well as the explanation on its operational algorithm. Section 3 describes how to model DRED as a continuous-time system using fluid-flow approximation. In addition, the TCP congestion control mechanism, the analytical model of link propagation delay, and the modeling method of an entire network proposed in the literature<sup>11</sup> are briefly reviewed. Section 4 analyzes the steady state behavior of DRED using MATLAB/Simulink. The validity of the proposed analytical model of DRED is examined through simulation experiments. Lastly, Section 5 describes a summary of this paper and future works.

## 2. DRED (DYNAMIC RANDOM EARLY DETECTION)

### 2.1. Overview

DRED<sup>7</sup> was designed to solve the problem that the average queue length in RED strongly depends on the number of TCP connections in steady state. Steady state means the state where the packet arrival rate to the router balances its packet processing capacity so that the average queue length at the router does not fluctuate.

DRED controls the number of packets in the buffer of a router by randomly dropping packets using a control method based on a classical control theory, I (Integral) control.<sup>14</sup> The I control adjusts an input to the plant. This control input is calculated from the integrate of difference (control error) between the control target and the output from the plant. Therefore, as long as the cumulative error from past is non-zero, a control input is adjusted to balance it, even if the current control error is equal to zero. In DRED, the queue of the router, corresponds to the plant, whereas the packet drop rate corresponds to the control input.

With this control mechanism, DRED can stabilize the average queue length of the router around the predetermined target value independent of the number of TCP connections. Consequently, it improves the utilization of links and keeps the delay at the router constant independent of the number of TCP connections.

**Table 1.** Recommended setting of DRED control parameters

$\Delta t$	sampling interval	10	[packet]
$\alpha$	control gain	0.00005	
$\beta$	filter gain	0.002	
$T$	target queue length	half of buffer size	[packets]
$B$	buffer size	double of bandwidth delay product	[packets]
$L$	packet non-drop threshold	$0.8T$ or $0.9T$	[packets]

## 2.2. Control algorithm of DRED

DRED stabilizes the average queue length of a router around the predetermined target value by adjusting packet drop rate using the average queue length of the router as feedback information. Packet drop rate is recalculated every sampling interval  $\Delta t$ , which is one of control parameters of DRED. In other words, DRED recalculates packet drop rate each time after  $\Delta t$  packets arrive at the router.

The following describes how the  $n$ th packet drop rate is calculated. First, DRED sets the current queue length  $q(n)$  to the one sampled when the designated number of packets arrive at the buffer of the router. Then, it calculates the error  $e(n)$  between the current queue length  $q(n)$  and the target one  $T$ , a control parameter of DRED.

$$e(n) = q(n) - T \quad (1)$$

It also calculates the average error of queue length  $\hat{e}(n)$  based on  $e(n)$ .

$$\hat{e}(n) = (1 - \beta)\hat{e}(n - 1) + \beta e(n), \quad (2)$$

where the average error between the current queue length  $q(n)$  and the target queue length  $T$  is estimated using an exponential weighted moving average, which is a type of low-pass filter.  $\beta$  is a control parameter of DRED, and serves as weight in this equation. Using  $\hat{e}(n)$ , the packet drop rate  $p_d(n)$  is calculated as

$$p_d(n) = \min \left[ \max \left\{ p_d(n - 1) + \alpha \frac{\hat{e}(n)}{B}, 0 \right\}, \theta \right], \quad (3)$$

where  $\alpha$  is a control parameter of DRED, and a weight to determine the variation of packet drop rate.  $B$  is the buffer size of the router.  $\theta$  is the maximum packet drop rate. Finally, the packet drop rate  $p(n)$  that is actually applied is given by

$$p(n) = \begin{cases} p_d(n) & \text{if } q(n) \geq L \\ 0 & \text{otherwise} \end{cases}, \quad (4)$$

which compares the current queue length  $q(n)$  with the packet non-drop threshold,  $L$ , which is also a control parameter of DRED.

Table 1 shows the recommended values of the control parameters of DRED presented in the literature.<sup>7</sup>

## 3. MODELING BASED ON FLUID-FLOW APPROXIMATION

### 3.1. Overview

In this section, we model each network component as a continuous-time system using fluid-flow approximation. We start with the explanation of a modeling method of the DRED router. We then outline the models of the TCP congestion control mechanism, link propagation delay, and entire network, proposed in the literature.<sup>11</sup> Table 2 lists the definitions of the symbols used in the followings.

**Table 2.** Definition of symbols (constants and variables)

$R$	TCP round-trip time
$c$	processing speed of DRED router
$\Delta t$	sampling interval of DRED router
$\alpha$	control gain of DRED router
$\beta$	filter gain of DRED router
$B$	buffer size of DRED router
$T$	target queue length of DRED router
$x(t)$	input to the model (packet transfer rate)
$y(t)$	output from the model (packet transfer rate)
$w(t)$	TCP window size
$q(t)$	current queue length of DRED router
$e(t)$	error of queue length of DRED router
$\hat{e}(t)$	average error of queue length of DRED router
$p(t)$	packet drop rate of DRED router

### 3.2. Modeling of DRED router

We model a DRED router as a continuous-time system, of which both the input  $x(t)$  and the output  $y(t)$  are the packet transfer rate. For DRED, using fluid-flow approximation, we model the variation of the current queue length  $q(t)$ , the error between the current queue length and the target queue length,  $e(t)$ , the variation of the average error between the current queue length and the target queue length  $\hat{e}(t)$ , and the variation of the packet drop rate  $p(t)$  as follows:

$$\dot{q} = \begin{cases} x(t) - c & \text{if } q(t) > 0 \\ (x(t) - c)^+ & \text{if } q(t) = 0, \end{cases} \quad (5)$$

$$e(t) = q(t) - T, \quad (6)$$

$$\dot{\hat{e}} = -\beta c(\hat{e}(t) - e(t)), \quad (7)$$

$$\dot{p} = \frac{1}{\Delta t} \alpha \frac{\hat{e}(t)}{B}, \quad (8)$$

where  $(x)^+ \equiv \max(x, 0)$ .

Equation (5) defines the variation of the current queue length of a DRED router.  $x(t)$  is the packet transfer rate to the DRED router, whereas  $c$  is its processing speed of DRED router. Variation of the current queue length of the DRED router  $\dot{q}$  is given as the excess of the packet transfer rate over the processing speed of DRED router.

Equation (6) defines the error between the current queue length of the DRED router and the target queue length (Eq. (1)).  $T$  is the target queue length of the queue length, which is one of the control parameters.

Equation (7) defines the variation of the average error between the current queue length of the DRED router and the target queue length (Eq. (2)).  $\beta$  is a weight for exponential weighted moving average. The average error between the current queue length of the DRED router and the target queue length is calculated every sampling interval, using the exponential weighted moving average. Thus, variation of the average error between current queue length of the DRED router and the target queue length  $\dot{\hat{e}}$  is given by the difference between average error  $\hat{e}(t)$  and error  $e(t)$  multiplied by both the weight for exponential weighted moving average,  $\beta$ , and the processing capacity of the DRED router,  $c$ , which is the approximation of packet arrival rate  $x(t)$ .

Equation (8) defines the variation of packet drop rate of the DRED router.  $\alpha$  is a weight to determine the variation of packet drop rate. At the DRED router, the packet drop rate is recalculated every sampling interval,  $\Delta t$ . Therefore, an instantaneous variation of the packet drop rate  $\dot{p}$  is given by an actual variation of the packet drop rate divided by sampling interval  $\Delta t$ . Note that we assume  $q(t) \geq L$ .

Figure 1 displays a block diagram of our analytical model of DRED.

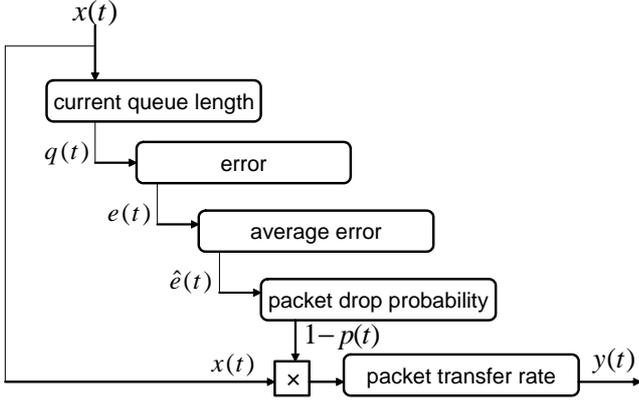


Figure 1. A block diagram for analytical model of DRED

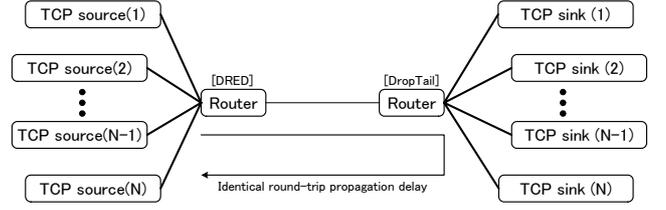


Figure 2. Network topology used in numerical example and simulation

### 3.3. Modeling of TCP congestion control mechanism

In the literature,<sup>11</sup> the TCP congestion control mechanism is modeled as follows:

$$\dot{y} = \frac{x(t)}{y(t)R^2} - \frac{2}{3}y(t) \{y(t-R) - x(t)\}, \quad (9)$$

where  $x(t)$  is the arrival rate for ACK packets,  $y(t)$  is the packet transfer rate, and  $R$  is the TCP round-trip time. The literature<sup>11</sup> assumes in the modeling that all of the TCP connections always have data to transfer, TCP round-trip time  $R$  is constant and only TCP slow start phase is negligible. That is, TCP slow start phase, TCP timeout mechanism and so on are out of the scope.

### 3.4. Modeling of link propagation delay

The literature<sup>11</sup> models link propagation delay as follows:

$$y(t) = x(t - \tau), \quad (10)$$

where  $x(t)$  and  $y(t)$  denote the transfer rates of packets which are input to and output from the link, respectively, and  $\tau$  denotes the link propagation delay.

### 3.5. Modeling of an entire network

We lastly outline the modeling of an entire network as suggested in the literature.<sup>11</sup> In the literature,<sup>11</sup> a model of an entire network is built by interconnecting models of the TCP congestion control mechanism, a RED router, and the link propagation delay. We substitute in this paper a model of a DRED router for that of a RED one to model an entire network.

If DRED has several input links, aggregate the flows on these links. In particular, a confluence of the flows can be expressed by the sum of the packet rate on each link. Let  $x_i(t)$  ( $1 \leq i \leq N$ ) denote the packet transfer rate of each flow and  $y(t)$  denote the sum of the packet transfer rates. Then,

$$y(t) = \sum_{i=1}^N x_i(t). \quad (11)$$

If DRED has several output links, assign the output of DRED to the flows on these links. Assignment is done by splitting a single packet transfer into  $N$  packet transfers, where  $N$  is the number of output links. Let  $x(t)$  denote the flow before the split,  $y_i(t)$  ( $1 \leq i \leq N$ ) each flow after the split, and  $f_i(t)$  ( $1 \leq i \leq N$ ) the share of each flow. Then,

$$y_i(t) = f_i(t)x(t). \quad (12)$$

The share of the flow,  $f_i(t)$  ( $1 \leq i \leq N$ ), is given by

$$f_i(t) = \frac{\sum_{s' \in S_{L(i)}} z_{s'}(t)}{\sum_{s \in S_R} z_s(t)}, \quad (13)$$

where  $S_{L(i)}$  is a set of TCP flows passing through the link  $i$ ,  $S_R$  is a set of TCP flows passing through the DRED router  $R$ , and  $z_s(t)$  is the packet output rate of the TCP flow  $s$  at time  $t$ .

#### 4. NUMERICAL EXAMPLES AND SIMULATION

In this section, we show the impact of the variations in DRED control parameters such as the sampling interval on the performance of DRED with a numerical example of the steady state analysis of DRED. We also validate the proposed analytical model of DRED through the comparison of its analysis results with simulation results using ns-2. Both the current queue length and the packet drop probability in DRED are compared for this purpose.

The network model used for the numerical example and the simulations has a single DRED router with several TCP connections, of which the transfer rates are identical (Fig. 2). In Fig. 2, TCP source( $i$ ) represents a TCP sender, while TCP sink( $i$ ) represents a TCP receiver. That is, the TCP source( $i$ ) transfers packets to the TCP sink( $i$ ).  $1 \leq i \leq N$  is the identifier of the TCP sources/sinks. Incidentally, the bandwidth of the link between TCP sources/sinks and the router is sufficiently larger than that between two routers in Fig. 2. This means that the link between two routers is the bottleneck.

We firstly built each of the continuous-time models of the DRED router, the TCP congestion control mechanism, and the link propagation delay as described in Section 3, using MATLAB/Simulink<sup>12</sup> which is a control system CAD tool. We then built a model of an entire network as a continuous-time system by interconnecting the above continuous-time models. Next, we conducted numerical simulations of the continuous-time system of the entire network and measured the variations of the current queue length and the packet drop rate of the router, as well as their averages.

We also performed simulations of this network model using ns-2 (version 2.27). In the simulations, we assumed that all of the TCP senders in the network always have data to send. We ran simulations for 200 [s], and calculated the averages of the current queue length and the packet drop rate of the DRED router for the last 100 [s] simulations. In the simulations, the size of the packets transferred by TCP was fixed at 576 [byte].

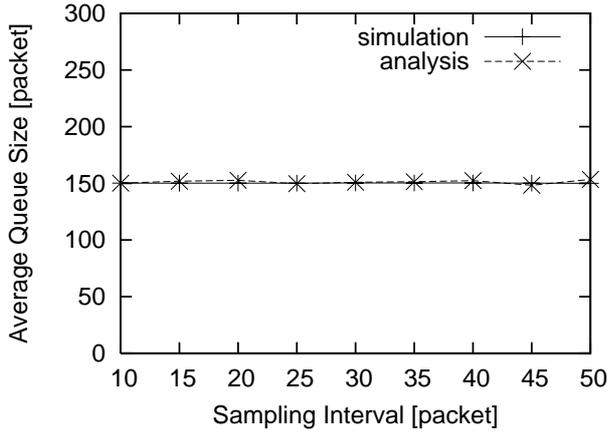
The following parameters were used in both the numerical example and the simulations unless stated otherwise: as system parameters, the number of TCP connections  $N = 30$ , bandwidth of the bottleneck link  $\mu = 2.50$  [packets/ms], and round-trip propagation delay of the TCP connections  $\tau = 50$  [ms]; and as control parameters of DRED, sampling interval  $\Delta t = 10$  [packets], control gain  $\alpha = 0.0005$ , filter gain  $\beta = 0.02$ , buffer size of the router  $B = 300$  [packets], target queue length  $T = 150$  [packets], and packet non-drop threshold  $L = 130$  [packets].

Both numerical analysis and simulation results are shown in Fig.3 through Fig.8. Figures 3 and 4 show the average queue length and the packet drop probability, respectively, of the DRED router in steady state when varying the sampling interval  $\Delta t$ . Figures 5 and 6 also show the average queue length and packet drop probability of the DRED router in steady state, when varying the number of TCP connections  $N$ . Figures 7 and 8 show the same metrics as above when varying the bandwidth of the bottleneck link  $\mu$ .

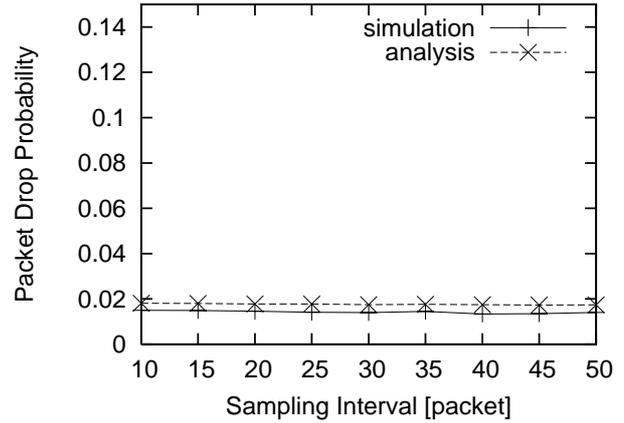
It can be seen from Figs. 3 and 4 that the results of the numerical analysis and the simulation almost completely correspond to each other. In both results, the current queue length and the packet drop probability stay constant, even if the sampling interval  $\Delta t$  changes. This is presumably because the sampling interval  $\Delta t$  is a control parameter that determines transient characteristics of DRED and has little impact on its steady state characteristics including these metrics.

Next, from Figs. 5 and 6, we note that the result of the numerical analysis is different from that of the simulation for the latter case, i.e., in terms of the packet drop probability, when the number of TCP connections is large (Fig. 6). This can be attributed to the following: as the number of TCP connections increases, the window size for each TCP connection becomes smaller. Then, the decline in the transfer rate due to TCP timeouts becomes more likely to occur. However, the decline in the transfer rate due to TCP timeouts was not incorporated in our analytical model of the TCP congestion control mechanism. Therefore, the result of the numerical analysis diverged from that of the simulation.

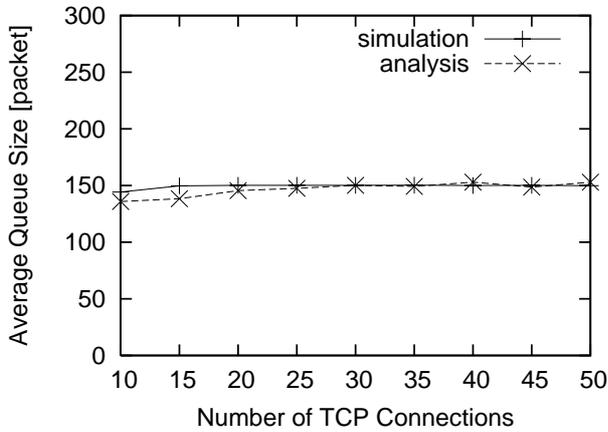
Lastly, in Figs. 7 and 8, the results of the numerical analysis and the simulation closely correspond to each other. This means that our approximate analytical method is valid for steady-state analysis. From these figures, which show the impact



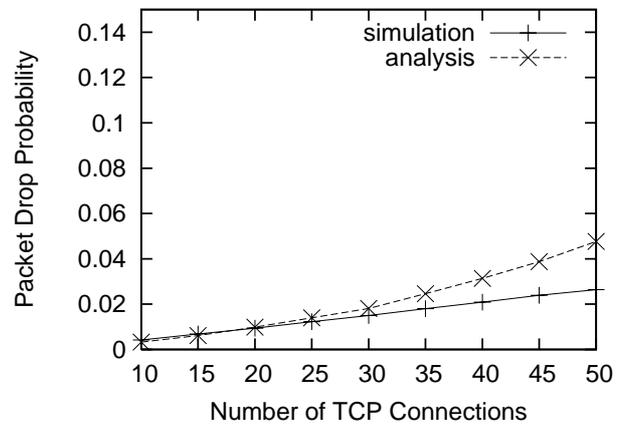
**Figure 3.** Average queue length of DRED in steady state (sampling interval  $\Delta t = 10 - 50$  [packet])



**Figure 4.** Packet drop probability of DRED in steady state (sampling interval  $\Delta t = 10 - 50$  [packet])



**Figure 5.** Average queue length of DRED in steady state (number of TCP connections  $N = 10 - 50$ )



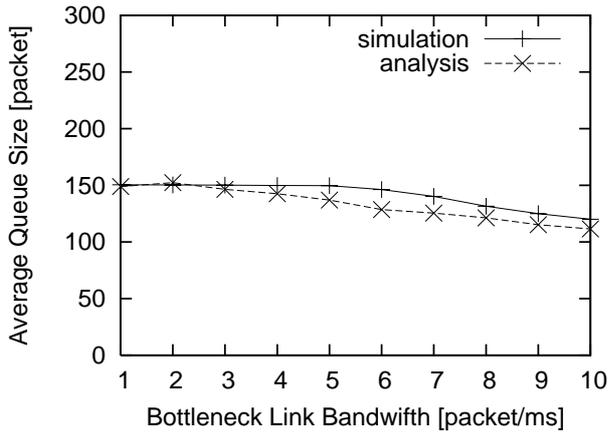
**Figure 6.** Packet drop probability of DRED in steady state (number of TCP connections  $N = 10 - 50$ )

of the bandwidth of the bottleneck link, we can also see that the queue length becomes smaller than the target value where the bottleneck link bandwidth is high. Further investigation into the causes generating the above results revealed that in both the numerical analysis and the simulation there were some cases where the packet drop probability was reduced to zero. Thus, if the bandwidth delay product of the network is significantly large, the packet drop probability becomes so small that DRED cannot stabilize the current queue length of the router around the target value.

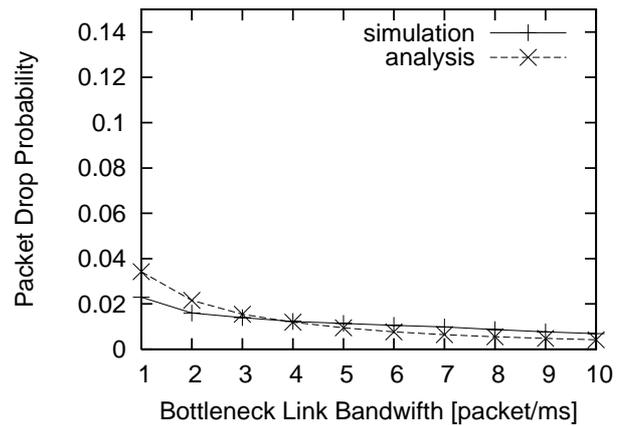
## 5. SUMMARY AND FUTURE WORKS

In this paper, we conducted a steady state analysis of the active queue management mechanism DRED using the analytical method proposed in the literature.<sup>11</sup> We modeled a DRED router as a continuous-time system with the use of fluid-flow approximation and thereby derived its average queue length and packet drop probability in steady state. We also validated the approximate analytical method through the comparison of the results of the steady state analysis using the proposed analytical model with those of the simulation.

Our future works will involve the modeling of components in a network other than DRED. For example, we believe that the active queue management mechanism REM and the PD controller based on control theory would be easily modeled by extending the analytical model of DRED. In addition, a wide range of performance evaluation will become possible



**Figure 7.** Average queue length of DRED in steady state (bottleneck link bandwidth  $\mu = 1 - 10$ )



**Figure 8.** Packet drop probability of DRED in steady state (bottleneck link bandwidth  $\mu = 1 - 10$ )

by applying control theory to analytical models of continuous-time system. For instance, numerical simulations using a control system CAD tool will enable steady state analysis as well as transient state analysis. The numerical example described in this paper is such an application to steady state analysis. As an example of transient state analysis, it will be possible to analyze the dynamics of the queue length of a DRED router.

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