Multivariate Analysis for Performance Evaluation of Active Queue Management Mechanisms in the Internet

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ABSTRACT

AQM (Active Queue Management) mechanism, which performs congestion control at a router for assisting the end-to-end congestion control mechanism of TCP, has been actively studied by many researchers. For instance, RED (Random Early Detection) is a representative AQM mechanism, which drops arriving packets with a probability being proportional to its average queue length. The RED router has four control parameters, and its effectiveness heavily depends on a choice of these control parameters. This is why many researches on the parameter tuning of RED control parameters have been performed. However, most of those studies have investigated the effect of RED control parameters on its performance from a small number of simulation results. In this paper, we therefore statistically analyze a great number of simulation results using the multivariate analysis. We quantitatively show the relation between RED control parameters and its performance.

Keywords: AQM (Active Queue Management) Mechanism, RED (Random Early Detection), Parameter Tuning, Multivariate Analysis

1. INTRODUCTION

In the last few years, AQM (Active Queue Management) mechanism, which supports the end-to-end congestion control mechanism of TCP, has been actively studied by many researchers. AQM mechanism controls the queue length of a router (i.e., the number of packets in the buffer) by actively dropping arriving packets. It has been reported that the AQM mechanism can solve several problems of conventional Drop-Tail routers[1].

RED (Random Early Detection) is the most popular AQM mechanism[2]. A router employing the RED mechanism is called a *RED router*. The RED router randomly drops arriving packets with a probability, which is calculated from four RED control parameters and its average queue length. The RED router does not distinguish TCP connections; i.e., it uses the same packet dropping discipline for all TCP connections. Hence, the implementation of the RED router is simple. However, the RED router has several drawbacks. In particular, it is known that effectiveness of the RED router is heavily dependent on a choice of control parameters[2, 3]. Since it is complicated to analyze both TCP and the RED router simultaneously, most researches on the RED router have been performed based on simulation experiments. For instance, simulation experiments are repeated by changing RED control parameters, and effects of RED control parameters on its response time are investigated in the past study[4]. In another study[5], the effect of each RED control parameter on its throughput is investigated from several simulation results. However, these simulation studies are simply based on a small number

of simulation results, and effects of RED control parameters on its performance have not been investigated in a systematic way.

In this paper, we therefore systematically analyze the performance of the RED router by applying the multi-variate analysis for a large number of simulation results. Namely, we first perform a great number of simulation experiments by changing the combination of RED control parameters. We then analyze thousands of simulation results statistically. In this paper, we investigate effects of RED control parameters on its performance by applying the multivariate analysis; i.e., one of several performance metrics (i.e., the average queue length, the throughput, and the packet loss probability) is chosen as a response variable, and RED control parameters are chosen as predictor variables. Of several multivariate analysis methods, we utilize the multiple regression analysis.

The rest of this paper is organized as follows. In Section 2, we briefly explain the RED router and its control parameters. In Section 3, overview of the multiple regression analysis is presented. In Section 4, we explain the simulation model, which will be used throughout this paper. In Section 5, the multivariate analysis is performed for thousands of simulation results obtained, and effects of RED control parameters on its performance is thoroughly investigated. Finally, in Section 6, we conclude this paper and discuss future works.

2. RED (RANDOM EARLY DETECTION) ALGORITHM

In this section, we briefly explain the operation algorithm of the RED router. Refer to another paper[2] for the details. The RED router randomly drops arriving packets with a probability, which is determined by its average queue length (i.e., the average number of packets in the router's buffer). Let q be the current queue length. When a packet arrives at the RED router, the estimated value of the average queue length, $\bar{q}$, is updated as

$$\bar{q} \leftarrow (1 - w_q)\bar{q} + w_q q \tag{1}$$

where w_q is a control parameter, which is the weight of a low pass filter. The RED router determines the packet loss probability p_b based on the average queue length $\bar{q}$ as

$$p_b = \begin{cases} 0 & \text{if } \bar{q} < min_{th} \\ 1 & \text{if } \bar{q} \geq max_{th} \\ max_p(\frac{\bar{q} - min_{th}}{max_{th} - min_{th}}) & \text{if } min_{th} \leq \bar{q} < max_{th} \end{cases} \tag{2}$$

where min_{th} is the minimum threshold, max_{th} is the maximum threshold, max_p is the maximum packet loss probability. These are control parameters of the RED router. Finally, the RED router randomly drops arriving packets with the probability p_a defined by

$$p_a = \frac{p_b}{1 - count \times p_b} \tag{3}$$

where $count$ is the number of packets that have arrived at the RED router since the last packet dropping. The RED router does not distinguish TCP connections, and drops all packets identically with the same packet loss probability p_a. Since the packet loss probability p_a is determined from RED control parameters (i.e., w_q, min_{th}, max_{th}, max_p), changing RED control parameters directly affects its performance metrics such as the average queue length.

3. MULTIPLE REGRESSION ANALYSIS

In this paper, we analyze the performance of the RED router using the multivariate analysis for thousands of simulation results. Of several multivariate analysis methods, we utilize a *multiple regression analysis*[6]. In what follows, we briefly explain the multiple regression analysis. Refer to the reference[6] for the details of the multiple regression analysis.

The multiple regression analysis is a method of statistically analyzing effects of *predictor variables* on a *response variable*[6]. Predictor variables are parameters that should affect the response variable, and the response variable is a parameter that should be changed when predictor variables are changed. When applying the multiple regression analysis, both predictor variables and the response variable must be chosen appropriately. If these variables are chosen inappropriately, the result of the multiple regression analysis would be meaningless although it seems to be plausible. For instance, the multiple regression analysis might indicate the correlation between predictor variables and the response variable, even though the response variable is independent of predictor variables. Thus, before performing the multiple regression analysis, it is necessary to check the correlation between these variables for choosing predictor variables and the response variable appropriately . In this paper, we therefore use a *pairwise scatter plot*[7], which is a set of scatter plots for each variable pair, for checking the correlation between variables.

In the multiple regression analysis, a *regression equation* representing the average relationship between the response variable and predictor variables is obtained from a set of measured predictor variables and the response variable. Provided that the response variable y has linearity regarding predictor variables, a regression equation for the response variable is given by linear combination of n predictor variables x_i.

$$y = \beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n + \epsilon \tag{4}$$

where the coefficient of a predictor variable, β_i, is called a *regression coefficient*, which indicates how much the predictor variable x_i affects the response variable y. In the above equation, ϵ is called a *residual*. We introduce $\hat{y}$ as the estimated value of the response variable from measured predictor variables and the regression equation, i.e., $\hat{y} \equiv y - \epsilon$. In the multiple regression analysis, by assuming that the mean of residuals is zero and that the distribution of residuals follows the normal distribution, β_i is determined from m sets of measured response variable and predictor variables. Once the regression equation is obtained appropriately, effects of predictor variables on the response variable can be investigated from regression coefficients β_i.

In the multiple regression analysis, the smaller the residual is, the better the accuracy of the regression equation is. As an index for measuring the accuracy of the regression equation, R^2 (multiple R-squared) is widely used in the multiple regression analysis. R^2 is defined from the ratio of residual variance to response variable's variance. More specifically, R^2 is defined as

$$R^2 = 1 - \frac{\sum_{i=1}^{m} \epsilon_i^2}{\sum_{i=1}^{m} (\hat{y}_i - \bar{y})^2} = \frac{\sum_{i=1}^{m} (y_i - \bar{y})^2}{\sum_{i=1}^{m} (\hat{y}_i - \bar{y})^2} \tag{5}$$

where $\bar{y}$ is the mean of m response variables. In the above equation, y_i, $\hat{y}_i$, and ϵ_i represent y, $\hat{y}$, and ϵ calculated from ith set of measured response variables and predictor variables. R^2 takes a value between 0 and 1, and it indicates how well the response variable is explained by predictor variables. If R^2 is large, it suggests that the response variable is well explained by predictor variables. Conversely, if R^2 is small, it suggests that the response variables cannot be explained by predictor variables.
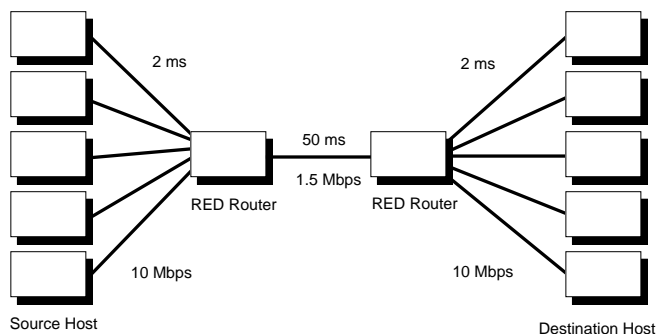
Figure 1. Simulation model

4. SIMULATION

Figure 1 illustrates the network model used in our simulation. There exists two RED routers and five homogeneous TCP connections. In this network model, the link between two RED routers is the bottleneck. We have run thousands of simulation experiments using this network model. Table 1 summarizes parameters used in our simulation.

Each simulation has been run for 30 [s], and the simulation result of the last 5 [s] is used for evaluating the RED performance. Namely, three performance measures of the RED router, the average queue length, the throughput, and the packet loss probability, are calculated from the simulation result of 25–30 [s]. Under these conditions, we have obtained thousands of simulation results while changing RED control parameters.

5. ANALYSIS RESULTS AND DISCUSSIONS

5.1. Packet Loss Probability

The pairwise scatter plot when the packet loss probability of the RED router is chosen as the response variable is shown in Fig. 2. This figure shows, for example, that there is almost linear relation between w_q and the packet loss probability, and that the relation between max_{th} and the packet loss probability is nonlinear. In the multiple regression analysis, it is assumed that the relation between predictor variables and the response variable is linear. If it is nonlinear, the multiple regression analysis cannot be applied. In such a case, it is necessary to perform some variable transformation. In what follows, for demonstrating effectiveness of variable transformation, we first show the result when variable transformation is not used.

Table 2 shows the result of the multiple regression analysis when RED control parameters are chosen as predictor variables and the packet loss probability is chosen as the response variable. In this table, a column

Table 1. Parameter values used in the simulation

bandwidth of bottleneck link	1. 5	[Mbit/s]
bandwidth of access link	10	[Mbit/s]
propagation delay of bottleneck link	50	[ms]
propagation delay of access link	2	[ms]
TCP packet size	1,000	[byte]
buffer size of RED router	100	[packet]

Table 2. Result of the multiple regression analysis on the packet loss probability

predictor variable	regression coefficient	standardized regression coefficient	t-value	P-value
(intercept)	4.63		70.94	0
w_q	-19.54	-0.08	-7.11	0
min_{th}	0.01	0.05	4.73	0
max_{th}	0.07	0.74	-58.32	0
max_p	-10.77	-0.10	-8.71	0
R^2	0.50			

named *regression coefficient* shows coefficients of predictor variables of the regression equation, and a column named *standardized regression coefficient* shows regression coefficients normalized by standard deviations of predictor variables and the response variable. Using these standardized regression coefficients, it becomes possible to compare effects of multiple predictor variables, which originally have different distributions. A column named *t-value* is the result of t-test, which investigates whether the distributions of residuals is changed by removing one of predictor variables from the regression equation. A column named *P-value* shows the probability that the distribution of residuals is the same when one of predictor variables is removed from the regression equation. A column named R^2 shows how much influence a predictor variable has on the response variable.

From Tab. 2, one can find that all P-values of predictor variables are zero. This suggests that from the multiple regression analysis, all predictor variables affect the response variable. However, R^2 is small (i.e., 0.50), which implies that the regression equation cannot predict the response variable so accurately. Hence, in what follows, we improves the accuracy of the regression equation by performing variable transformation. In general, an appropriate variable transformation method should be chosen from many transformation methods by observing how the pairwise scatter plot is affected by the chosen variable transformation method. From Fig. 2, it seems that a logarithmic transformation is appropriate. On the contrary, there is a paper[8] where the average queue length of the RED router is analytically derived, so that we can choose a variable transformation method based on this analytic result. Namely, in the paper,[8] equilibrium values of the TCP window size w^* and the average queue length of the RED router q^* in steady state are derived as

$$w^* = \sqrt{\frac{1}{4} + \frac{1}{3N}\left(\frac{max_{th} - min_{th}}{max_p(\overline{q}^* - min_{th})} + 1\right)} - \frac{1}{2} \tag{6}$$

$$q^* = N w^* - B \tau \tag{7}$$

where N is the number of TCP connections, B is the bandwidth of the RED router, and τ is the two-way propagation delay. Note that the analytic model used in the paper[8] is the same with our simulation model. As can be seen from Eq. (2), the packet loss probability p_b is determined by the average queue length. Moreover, by focusing on the first term of the right side of Eq. (6), it can be found that the average queue length is determined by the product of min_{th}, $max_{th} - min_{th}$, and max_p. In other words, the average queue length of the RED router is approximately expressed by the summation of logarithmically transformed variables: $\log min_{th}$, $\log(max_{th} - min_{th})$, and $\log max_p$. In the followings, we therefore use $\log w_q$, $\log min_{th}$, $\log(max_{th} - min_{th})$, and $\log max_p$ as predictor variables.

The pairwise scatter plot in this case is shown in Fig. 3. By performing variable transformation, linear relation can be observed between, for example, the predictor variable $\log(max_{th} - min_{th})$ and the packet loss
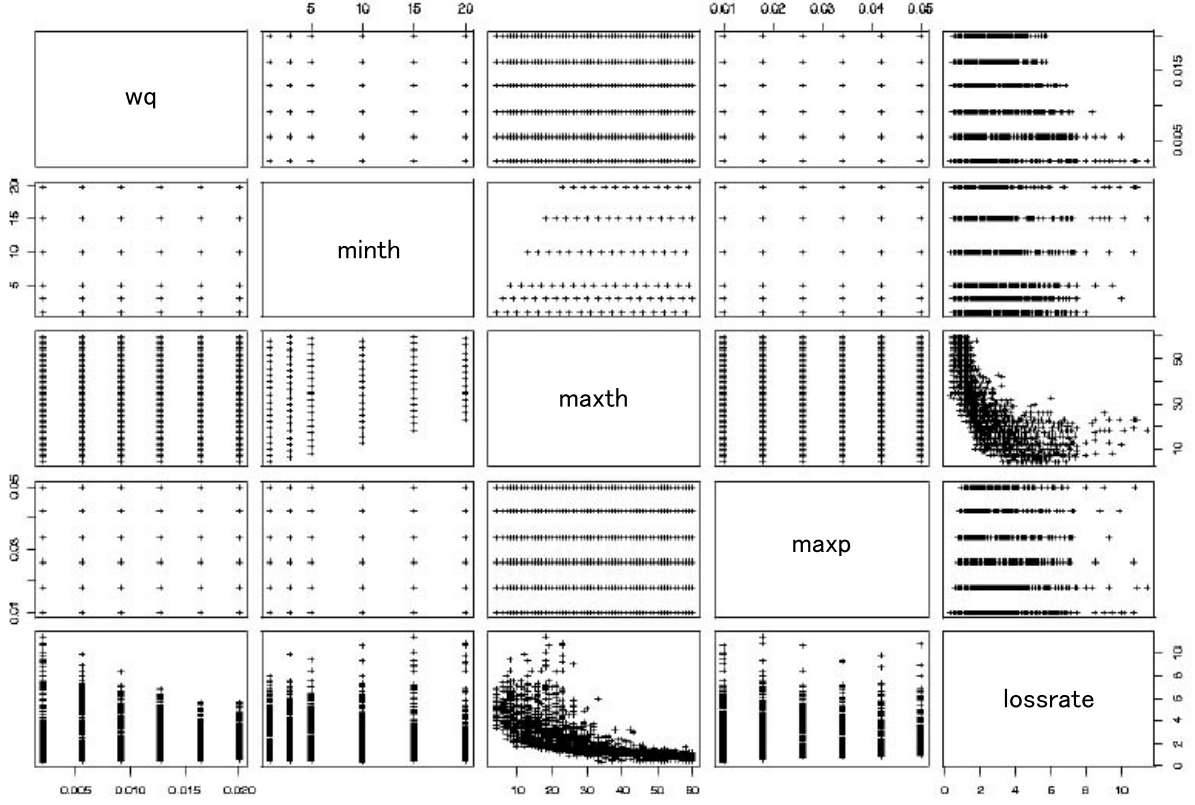
Figure 2. Pairwise scatter plot of control parameters and the packet loss probability

Table 3. Result of the multiple regression analysis on the packet loss probability (with variable transformation)

predictor variable	regression coefficient	standardized regression coefficient	t-value	P-value
(intercept)	5.04		35.55	0
$\log w_q$	-0.13	-0.03	-7.45	0
$\log min_{th}$	-0.28	-0.09	-20.14	0
$\log(max_{th} - min_{th})$	-1.47	-0.35	-81.67	0
$\log max_p$	0.32	0.05	-12.46	0
R^2	0.66			

probability. The result of the multiple regression analysis is shown in Tab. 3. It can be found that the value of R^2 is 0.66, which is larger than 0.50 of the previous case (see Tab. 2). Checking absolute values of standardized regression coefficients, one can find that the standardized regression coefficient of $\log(max_{th} - min_{th})$ is the largest (i.e., 0.35). The second largest is the standardized regression coefficient of $\log min_{th}$ (i.e., 0.09), which is much smaller than 0.35. This means that the packet loss probability of the RED router is mostly determined by $max_{th} - min_{th}$. On the other hand, it can be found that neither the maximum packet loss probability max_p nor the weight of the low pass filter w_q largely affect the packet loss probability. This is because operation of the RED router becomes unstable when $max_{th} - min_{th}$ is too small,[8] and many packet losses occur at the RED router regardless of the maximum packet loss probability max_p.
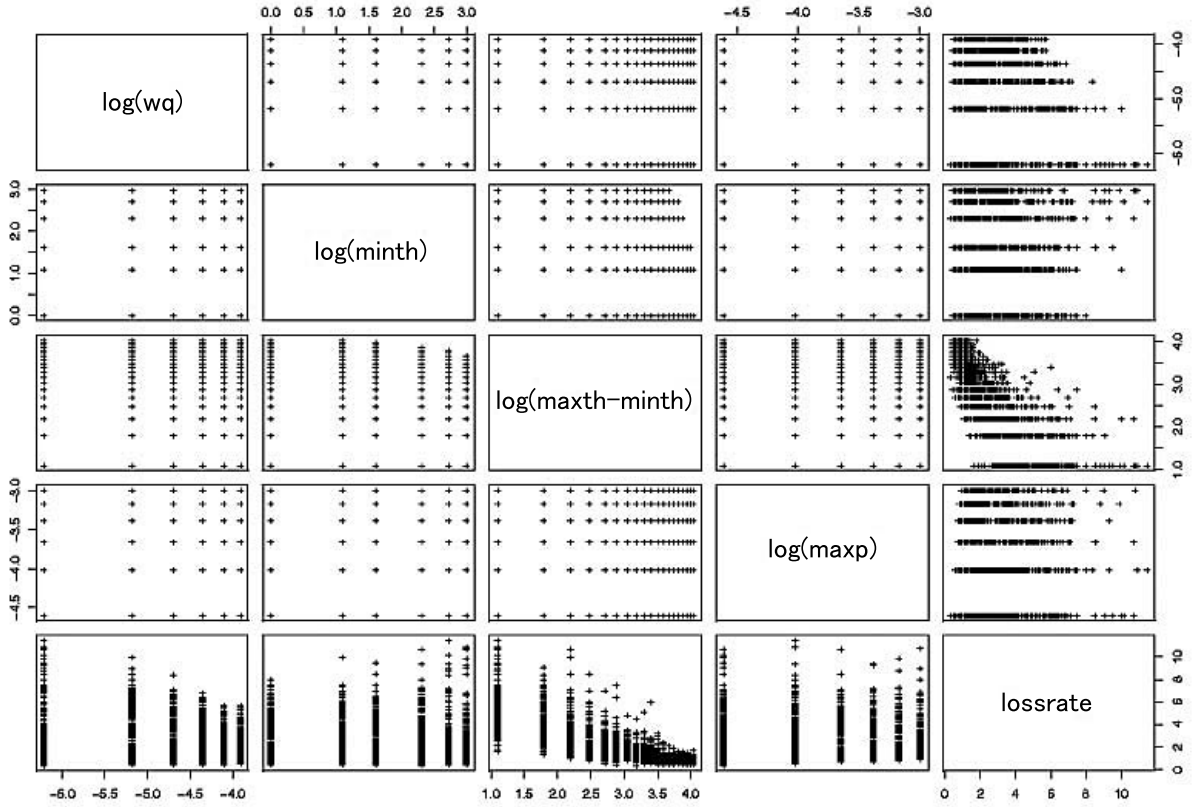
Figure 3. Pairwise scatter plot of control parameters and the packet loss probability (with variable transformation)

5.2. Average Queue Length

We next show the analysis result when the average queue length of the RED router is chosen as the response variable. In what follows, because of space limitation, we only show the result with logarithmically transformed predictor variables. The pairwise scatter plot when the average queue length is chosen as the response variable is shown in Fig. 4. As can be seen from this figure, there exists almost linear relation between RED control parameters and the average queue length. Table 4 shows the result of the multiple regression analysis. One can find that the value of R^2 is large (i.e., 0.88), which indicates that the accuracy of the regression equation is good. By focusing on absolute values of standardized regression coefficients, it can be found that max_{th} is the largest (i.e., 0.84). Absolute values of max_p (-0.33), min_{th} (0.14), and w_q (-0.02) become small in this order. This suggests that, at least in our simulation configuration, the maximum threshold max_{th} is the dominant factor. This phenomenon can be explained as follows. As can be seen from Eq. (2), the RED router discards all arriving packets when the average queue length is larger than max_{th}. Thus, the average queue length of the RED router is mostly upper-limited by the maximum threshold max_{th}, which explains why max_{th} is the dominant factor of the average queue length.

5.3. Throughput

The pairwise scatter plot when the throughput of the RED router is chosen as the response variable is shown in Fig. 5. The result of the multiple regression analysis is shown in Tab. 5. In this case, logarithmically transformed
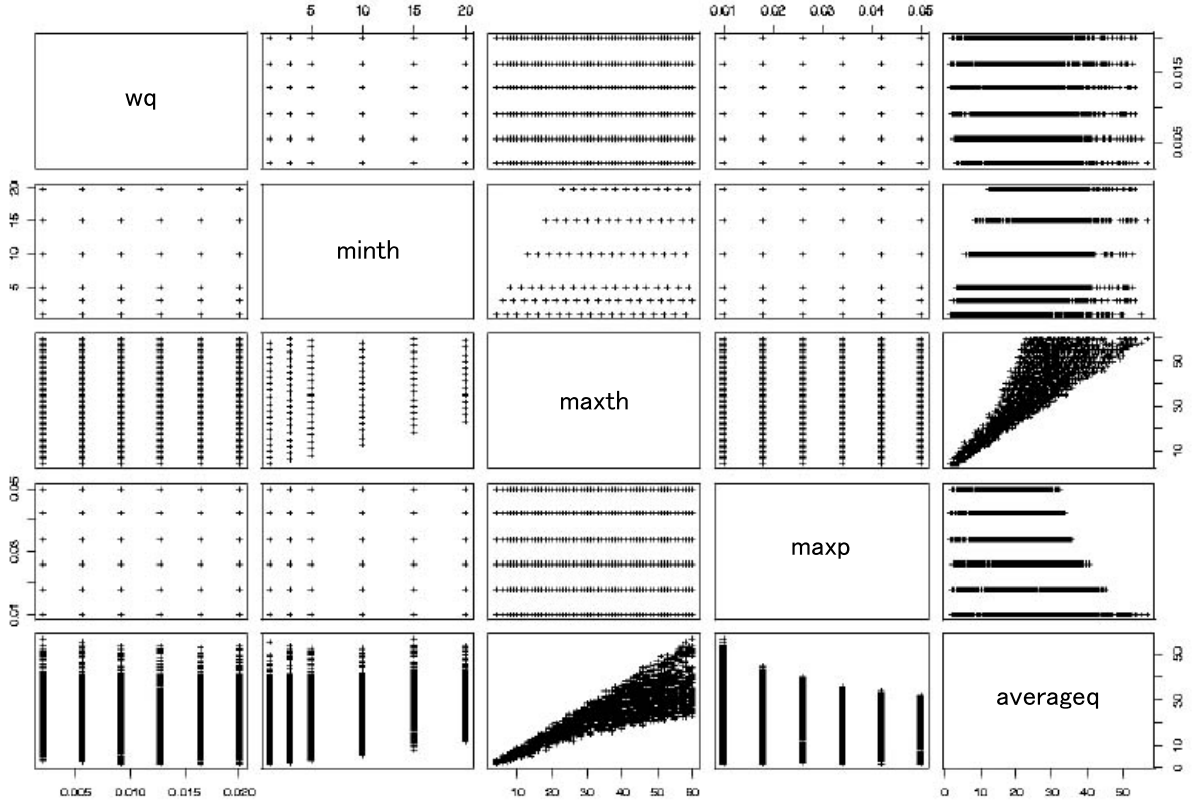
Figure 4. Pairwise scatter plot of control parameters and the average queue length

Table 4. Result of the multiple regression analysis on the average queue length

predictor variable	regression coefficient	standardized regression coefficient	t-value	P-value
(intercept)	9.75		41.87	0
w_q	-39.01	-0.02	-3.98	0
min_{th}	0.22	0.14	9.37	0
max_{th}	0.57	0.84	138.74	0
max_p	-249.44	-0.33	-56.56	0
R^2	0.88			

variables, $\log min_{th}$, $\log(max_{th} - min_{th})$, $\log max_p$, and $\log w_q$ are used as predictor variables. One can find that the value of R^2 is small (i.e., 0.36), which suggests that the accuracy of the regression equation is not good. On the contrary, by comparing absolute values of standardized regression coefficients, one can find that $\log(max_{th} - min_{th})$ is the largest (i.e., 0.26). This indicates that $max_{th} - min_{th}$ has the largest effect on the throughput of the RED router. This phenomenon is in agreement with the result of the multiple regression analysis on the packet loss probability (see Tab. 3).
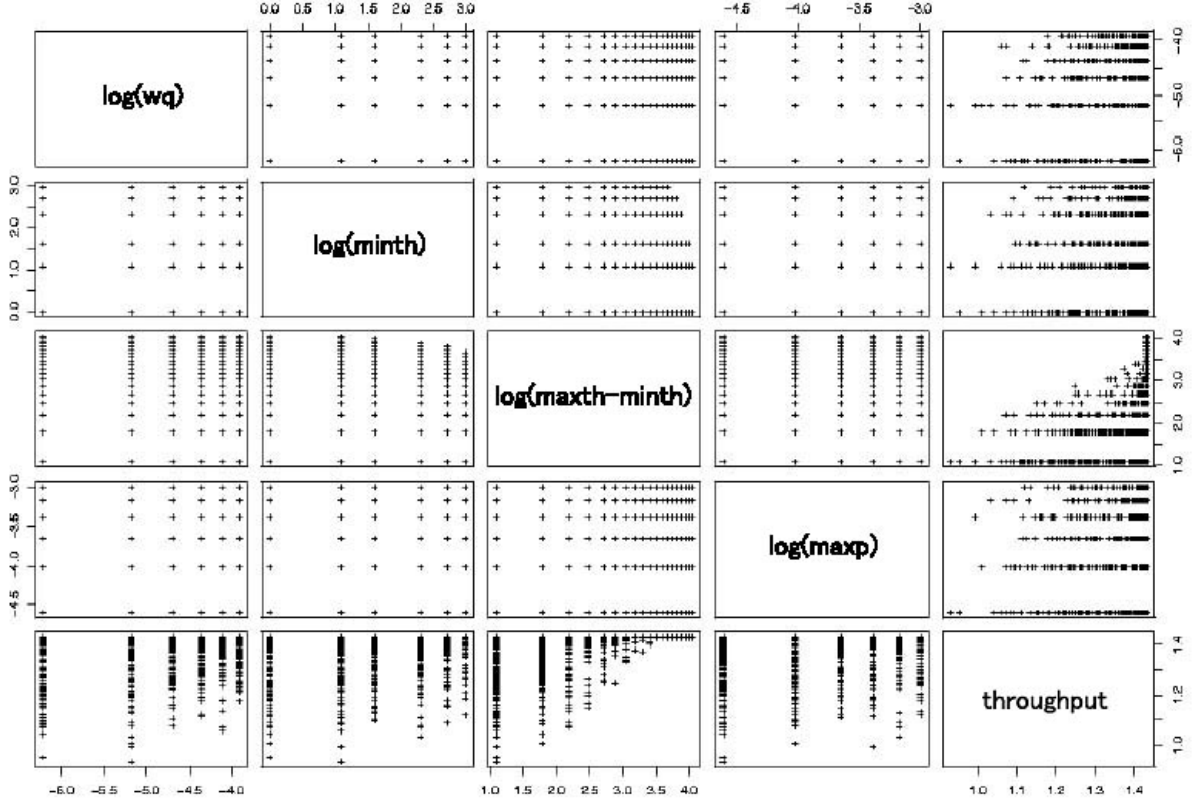
Figure 5. Pairwise scatter plot of control parameters and the throughput (with variable transformation)

Table 5. Result of the multiple regression analysis on the throughput (with variable transformation)

predictor variable	regression coefficient	standardized regression coefficient	t-value	P-value
(intercept)	1.35		194.28	0
$\log w_q$	0.006	0.04	6.85	0
$\log min_{th}$	0.01	0.09	13.84	0
$\log(max_{th} - min_{th})$	0.04	0.26	43.09	0
$\log max_p$	0.01	0.05	9.00	0
R^2	0.36			

6. CONCLUSION

In this paper, we have performed the multivariate analysis for thousands of simulation results, and have quantitatively shown how RED control parameters affect its performance. We have found (1) both the packet loss probability and the throughput of the RED router are largely affected by control parameters, $max_{th} - min_{th}$, min_{th}, max_p, and w_q, in this order. (2) the average queue length of the RED router is largely affected by control parameters, max_{th}, max_p, min_{th}, and w_q, in this order. (3) the control parameter w_q has little impact on the steady state performance of the RED router. These findings are in agreement with those of other simulation studies of the RED router, and it therefore proves the validity and effectiveness of the performance evaluation

using the multivariate analysis.

We are currently investigating effects of various system parameters (e.g., the number of TCP connections, the propagation delay, the bandwidth and the buffer size of the RED router) as well as RED control parameters on its performance using the multivariate analysis. Namely, by selecting system parameters as predictor variables and selecting one of performance metrics of the RED router as a response variable, the multiple regression analysis is performed for thousands of simulation results. Note that applicability of the performance evaluation method using the multivariate analysis is not limited to the RED router. We will therefore apply the multivariate analysis method to other traffic control schemes including other AQM mechanisms.

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